NORTHERN BEACHES SECONDARY COLLEGE
MANLY CAMPUS

MATHEMATICS EXTENSION 1

2024 Year 12 Course Assessment Task 4 – Trial HSC
Tuesday, 13 August 2024

General instructions

- Working time: 2 hours
  (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt all questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided on page 2

SECTION II

- Commence each new question in a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

- NESA reference sheet is provided
- Table of z-scores is provided on page 15

NESA STUDENT #: .... **# BOOKLETS USED:** ....

Marker's use only

QUESTION	1-10	11	12	13	14	Total	%
MARKS	$\frac{\square}{10}$	$\frac{\square}{15}$	$\frac{\square}{15}$	$\frac{\square}{15}$	$\frac{\square}{15}$	$\frac{\square}{70}$	

Section I

10 marks
Attempt Questions 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided.

Questions **Marks**

1. The success rate of a particular Bernoulli trial is 0.29.

   What is the variance of this trial? **1**

   (A) 0.2059

   (B) 0.2149

   (C) 0.1927

   (D) 0.3299

2. Evaluate the definite integral $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-4x^2}}\,dx$ **1**

   (A) $\frac{\pi}{2}$

   (B) $\frac{\pi}{3}$

   (C) $\frac{\pi}{4}$

   (D) $\frac{\pi}{6}$

3. 6 adults and 4 children need to be seated at a circular table.

   How many arrangements exist if the children must sit together? **1**

   (A) 120960

   (B) 17280

   (C) 362880

   (D) 30240

4. Given α, β and γ are the roots of the equation $4x^3 + 5x^2 + 4x - 1 = 0$,

$\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ is equal to: **1**

(A) -4

(B) 4

(C) -5

(D) 5

5. Which of the following is a correct expression for the primitive of $\int \cos^3 x \; dx$,

using the substitution $u = \sin x$? **1**

(A) $\sin x - \frac{1}{3}\sin^3 x + c$

(B) $\sin x - \frac{1}{3}\cos^3 x + c$

(C) $\cos x - \frac{1}{3}\sin^3 x + c$

(D) $\cos x - \frac{1}{3}\cos^3 x + c$

6. The scalar projection of vector $\underset{\sim}{\mathbf{u}}$ in the direction of vector $\underset{\sim}{\mathbf{v}}$ is -4.

If $\underset{\sim}{\mathbf{v}} = -\sqrt{3}\ \underset{\sim}{\mathbf{i}}$, the vector projection of $\underset{\sim}{\mathbf{u}}$ in the direction of $\underset{\sim}{\mathbf{v}}$ is: **1**

(A) $-4\underset{\sim}{\mathbf{i}}$

(B) $-3\underset{\sim}{\mathbf{i}}$

(C) $3\underset{\sim}{\mathbf{i}}$

(D) $4\underset{\sim}{\mathbf{i}}$

7. Consider the vectors $\underset{\sim}{\mathbf{a}} = -\operatorname{cosec}\theta\ \underset{\sim}{\mathbf{i}} + \sqrt{3}\ \underset{\sim}{\mathbf{j}}$ and $\underset{\sim}{\mathbf{b}} = \cos\theta\ \underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{j}}$.

If $\underset{\sim}{\mathbf{a}}$ is perpendicular to $\underset{\sim}{\mathbf{b}}$, then possible values for θ are: **1**

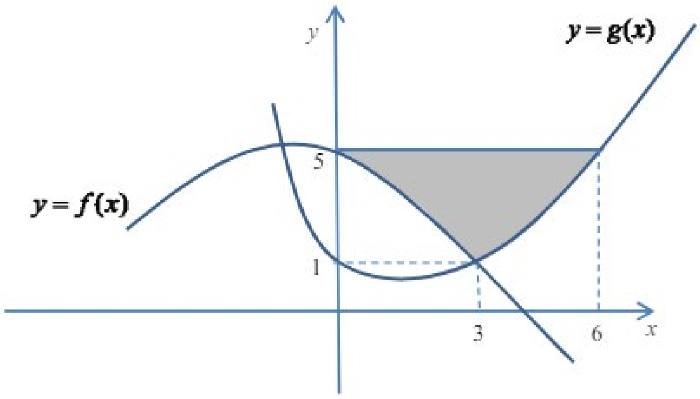
(A) $\dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$

(B) $\dfrac{\pi}{3}$ and $\dfrac{4\pi}{3}$

(C) $\dfrac{5\pi}{6}$ and $\dfrac{11\pi}{6}$

(D) $\dfrac{2\pi}{3}$ and $\dfrac{5\pi}{3}$

8. Consider the graphs of the two functions $y = f(x)$ and $y = g(x)$ below.



Which of the following definite integrals is equal to the area of the shaded region? **1**

(A) $\displaystyle\int_1^5 (g(x) - f(x))\ dx$

(B) $\displaystyle\int_1^5 (g(x) - g(x))\ dx$

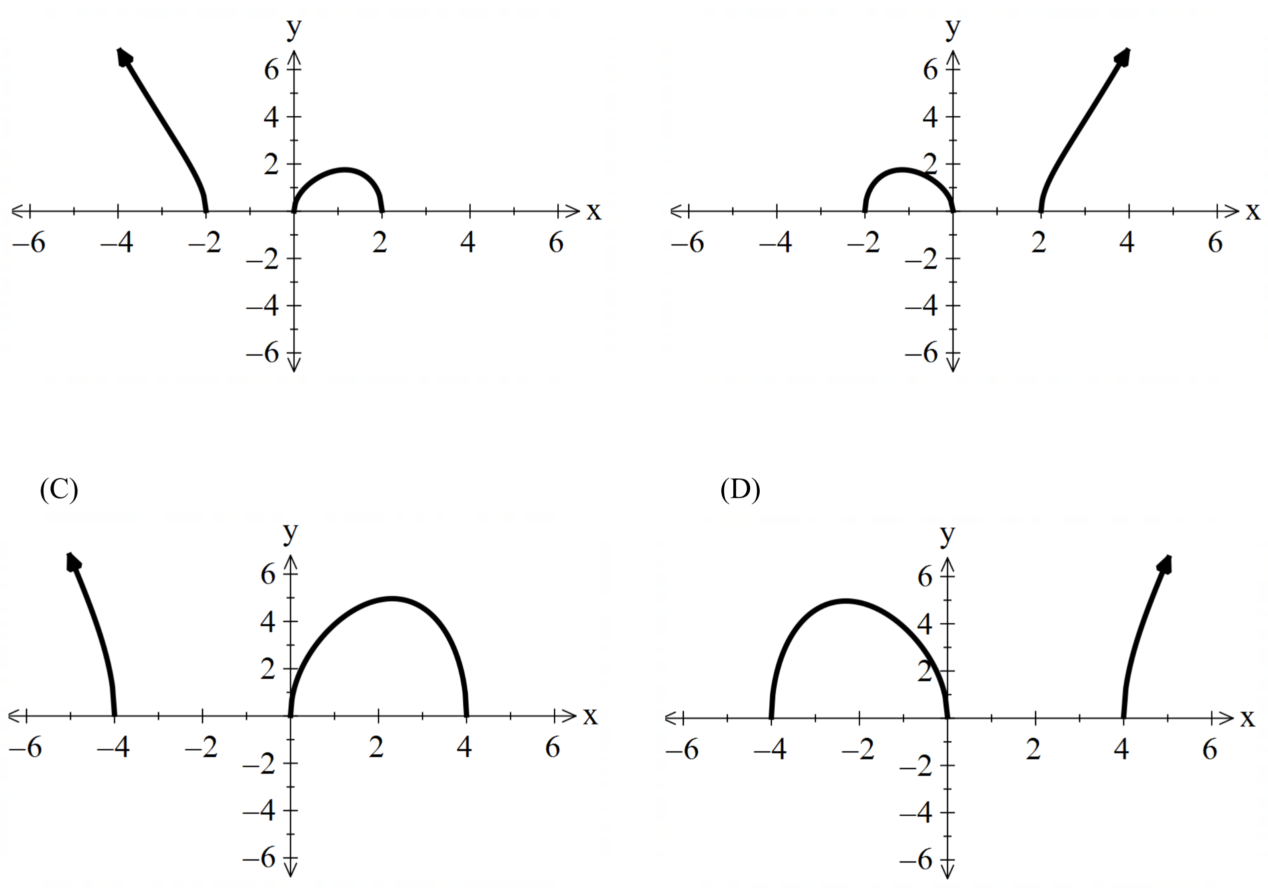
(C) $\displaystyle\int_0^3 (5 - g(x))\ dx + \int_3^6 (5 - f(x))\ dx$

(D) $\displaystyle\int_0^3 (5 - f(x))\ dx + \int_3^6 (5 - g(x))\ dx$

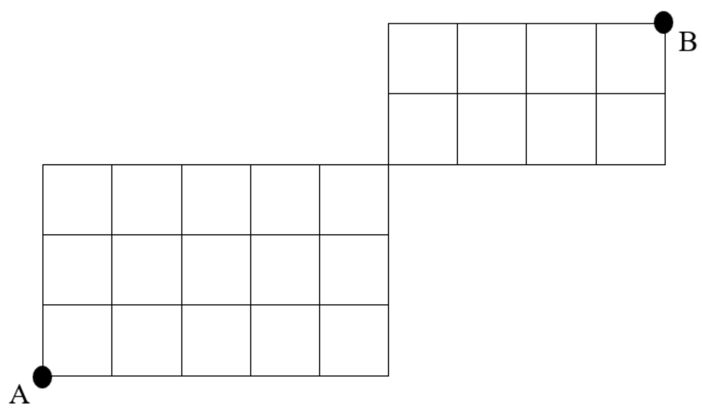
9. The graph of $y = \sqrt{x^3 - 4x}$ could be: **1**

(A) (B)

(C) (D)



10. Moving only right or up, how many different paths exist to get from Point A to Point B? **1**



(A) 45
(B) 840
(C) 480480
(D) 16!

End of Section I

Section II

60 marks
Attempt Questions 11 to 14
Allow approximately 1 hour 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet Marks

(a) What is the minimum number of people required in a group so that four of them are guaranteed to share the same birth month? **1**

(b) Let $\underset{\sim}{\mathbf{i}}$ be a unit vector pointing east and let $\underset{\sim}{\mathbf{j}}$ be a unit vector pointing north.

A group of hikers travels 5 km in the direction south $30°$ west and then north for 10 km.

Determine the exact position vector $\underset{\sim}{\mathbf{a}}$ of the group of hikers with respect to the starting point, in terms of $\underset{\sim}{\mathbf{i}}$ and $\underset{\sim}{\mathbf{j}}$. **3**

(c) Let X be a discrete random variable with binomial distribution $X \sim Bin(n,p)$.

The mean and the standard deviation of this distribution are equal.

Given that $0 < p < 1$, calculate the smallest number of trials, n, such that $p \leq 0.01$. **2**

(d) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{12}} \sin^2 4x \ dx$ **2**

Question 11 continues on the next page

Question 11 (continued)

(e) Let $P(x) = (x + 1)(x - 4)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x - 4$.

When $P(x)$ is divided by $x + 1$ the remainder is 15.

(i) Find the values of a and b **2**

(ii) Find the remainder when $P(x)$ is divided by $(x + 1)(x - 4)$ **1**

(f) On a 36°C day, Xiao takes an ice cream out of the freezer. The freezer is set at an optimal temperature of -18 °C. After one minute, the temperature of the ice cream is -10 °C.

The differential equation $\frac{dT}{dt} = -k(T - 36)$ models this situation where T is the temperature of the ice cream and t is the time in minutes out of the freezer.

(i) Show that the equation $T = 36 + Ae^{-kt}$, where A is a constant, satisfies the differential equation provided, **1**

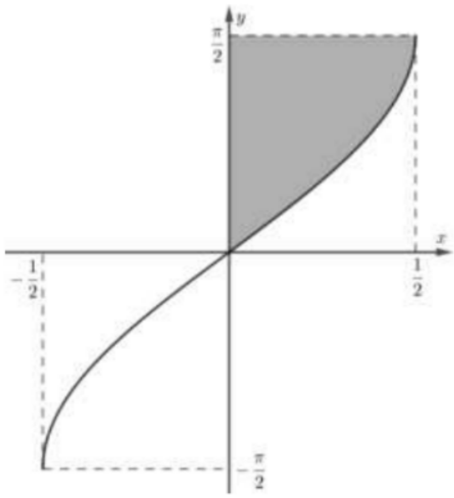
(ii) It is known that ice cream begins to melt at 0 °C. After how many minutes will the ice cream begin to melt? Give your answer to one decimal place. **3**

End of Question 11

Question 12 (15 marks) Start a new writing booklet Marks

(a) Find the number of ways that the letters of the word TRIANGLE can be arranged in a line such that the vowels are all next to each other but the consonants are not all next to each other **2**

(b) The diagram below shows the graph of $y = \sin^{-1} 2x$.



(i) Find the shaded area that is bounded by the curve, the y axis and the line $y = \frac{\pi}{2}$ **2**

(ii) If this area is rotated about the y axis, find the volume of the solid thus formed. **3**

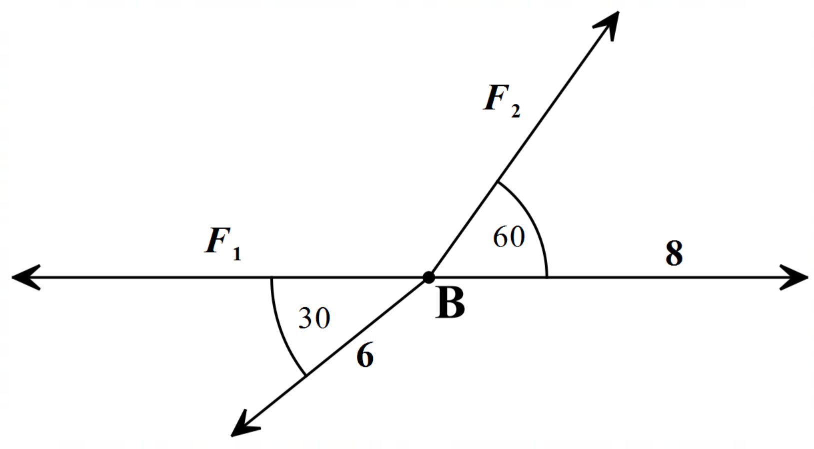
(c) Solve $\dfrac{1}{2x - x^2} \geq 1$ **3**

(d) Evaluate $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \frac{\sin(\tan^{-1} x)}{1 + x^2}\, dx$ using the substitution $u = \tan^{-1} x$ **2**

Question 12 continues on the next page

Question 12 (continued)

(e) The diagram below shows a stationary body, B, acted on by four forces.



Calculate the value of F_1. **3**

End of Question 12

Question 13 (15 marks) Start a new writing booklet **Marks**

(a) Consider the vectors $\underset{\sim}{\mathbf{a}} = x\,\underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{j}}$, $\underset{\sim}{\mathbf{b}} = \underset{\sim}{\mathbf{i}} - \underset{\sim}{\mathbf{j}}$ and $\underset{\sim}{\mathbf{c}} = \underset{\sim}{\mathbf{i}} + x\,\underset{\sim}{\mathbf{j}}$.

Given that θ is the angle between $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{\mathbf{b}}$, and φ is the angle between $\underset{\sim}{\mathbf{b}}$ and $\underset{\sim}{\mathbf{c}}$,

show that $\cos\theta\,\cos\varphi = \dfrac{-(x-1)^2}{2(1+x^2)}$ **2**

(b) For random samples of five employees at a tech company, $\hat{p}$ is the random variable that represents the proportion who work remotely.

Given that $P(\hat{p} = 0) = \dfrac{32}{243}$, find $P(\hat{p} > 0.6)$, correct to four decimal places. **3**

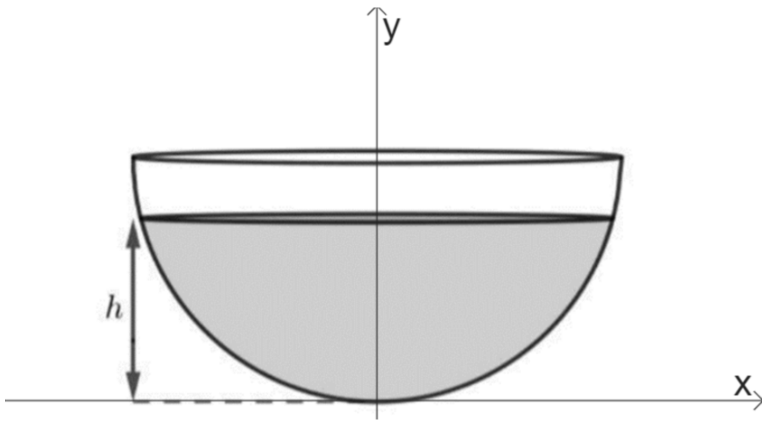
(c) Find the particular solution to the differential equation $\dfrac{dy}{dx} = \sqrt{16 - 3y^2}$ that passes through the point (0,2). **3**

(d) Find a simplified exact value of $\sin\left(\dfrac{\pi}{8}\right)$ **3**

Question 13 continues on the next page

Question 13 (continued)

(e) A hollow sphere with a diameter of 20m is cut in half and its top removed. Water is poured into the remaining half of the sphere at a constant rate of $20\ m^3$/minute. After 1 minute, the water level is h m above the base of the semi-sphere.



(i) Show that the curved edge of the sphere can be defined by the equation $x^2 = 20y - y^2$. **1**

(ii) Find an expression for the volume of the water in terms of h. **2**

(iii) Find the rate of change of the water level when the water is 6m above the base. **1**

End of Question 13

Question 14 (15 marks) Start a new writing booklet Marks

(a) Given that $f(x) = \dfrac{2}{x^2 + 1}$, find the value of $f^{-1}(f(N))$ for $N < 0$ **2**

(b) A function f has the rule $f(x) = xe^{2x}$.

Use mathematical induction to prove that $f^n(x) = (2^n x + n\, 2^{n-1})e^{2x}$ for $n > 0$, where $f^n(x)$ represents the nth derivative of $f(x)$. That is $f(x)$ has been differentiated n times. **3**

(c) Solve $\sin 3x = \sin 2x$ for $0 \le x \le 2\pi$ **3**

(d) Doctors are studying the resting heart rate of adults in the town of Mathsland.

Resting heart rate is measured in beats per minute (bpm).

The doctors consider a person to have a slow heart rate if the person's resting heart rate is less than 60 bpm. The probability that a randomly chosen Mathsland adult has a slow heart rate is 0.16.

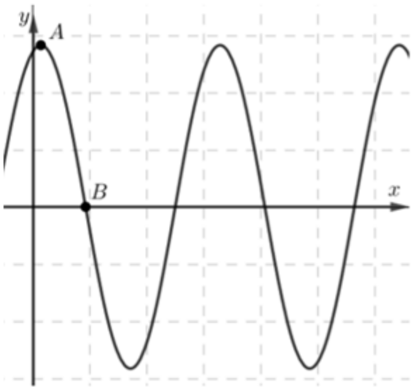
For random samples of 100 Mathsland adults, $\hat{p}$ is the random variable that represents the proportion of people who have a slow heart rate.

Find the probability that $\hat{p}$ is greater than 10%, correct to three decimal places. **3**

Question 14 continues on the next page

Question 14 (continued)

(e) The graph of $y = \sqrt{3}\,(\cos 2x + \sin 2x) - (\sin 2x - \cos 2x)$ is shown.



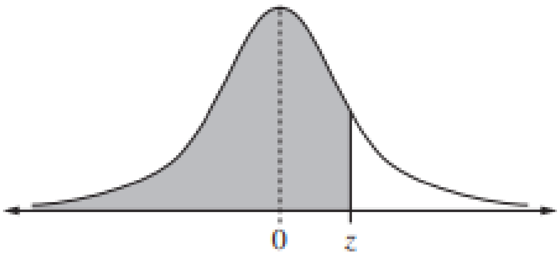
(i) Find the coordinates of A and B as labelled on the graph, given A is the maximum point on the graph and B is where the graph intersects the x-axis, without using calculus. **2**

(ii) Hence or otherwise, solve

$$\sqrt{3}\,(\cos 2x + \sin 2x) - (\sin 2x - \cos 2x) = 2\sqrt{2} \text{ for } 0 \le x \le 2\pi$$ **2**

End of paper

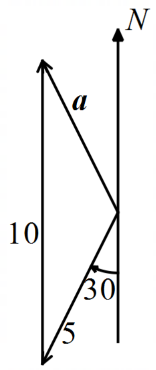
Table of values $P(Z \le z)$ for the normal distribution $N(0, 1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

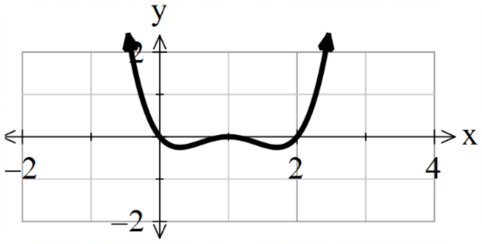
Math Extension 1 2024 Trial Solutions and Marking Criteria

Question	Solution	criteria
1	$\text{Var}(x) = p(1-p)$ $= 0.29(1-0.29)$ $= 0.2059$	A
2	$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-4x^2}}dx = \frac{1}{2}\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-(2x)^2}}dx$ $= \frac{1}{2}\left[\sin^{-1}2x\right]_0^{\frac{1}{2}}$ $= \frac{1}{2}\left(\frac{\pi}{2}-0\right)$ $= \frac{\pi}{4}$	C
3	First, consider the four children as a bundle so that with the other 6 adults there are initially 7 entities to be arranged in a circle. The number of ways to arrange 7 entities in a circle is (7 − 1)! = 6!. Then, the number of ways to arrange the four children in that bundle (in a line) is 4!. By the multiplication rule of counting, the number of arrangements is 6! × 4! = 17,280	B
4	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ $= \frac{\frac{c}{a}}{-\frac{d}{a}}$ $= 4$	B
5	$u = \sin x$ $du = \cos x\ dx$ $u^2 = \sin^2 x$ so $\int \cos^3 x dx = \int 1 - u^2$ du $= u - \frac{u^3}{3} + c$ $= \sin x - \frac{\sin^3 x}{3} + c$	A
6	Vector projection has magnitude 4 in the opposite direction to u, therefore 4i	D

7	$-cosec\theta cos\theta + \sqrt{3} = 0$ $-cot\vartheta = -\sqrt{3}$ $cot\theta = \sqrt{3}$ $tan\theta = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$	A
8	2 separate integrals with limits 0 to 3 and 3 to 6	D
9		B?
10	The diagram can be split into two separate rectangles as they only connect at a corner. In the first rectangle, there are a total of 5 rights and 3 ups required to get to the upper right corner. This can be done in 8!5!3! ways. In the second rectangle, there are a total of 4 rights and 2 ups required to get to the upper right corner. This can be done in 6!4!2! ways. The total number of paths will be 8!5!3!×6!4!2!=840.	B
11a	By the generalised pigeon-hole principle: $3 \times 12 + 1 = 37$	1 mark correct solution,
11b	$\vec{\mathbf{a}} = -5\sin(30)\vec{\mathbf{i}} + (10 - 5\cos(30))\vec{\mathbf{j}}$ $= -\frac{5}{2}\vec{\mathbf{i}} + \frac{20 - 5\sqrt{3}}{2}\vec{\mathbf{j}}$  	3 marks- correct solution. 2 marks- correct solution in incorrect form. 1 mark - correct diagram.

11c	$n\,p = \sqrt{npq}$ $n^2\,p^2 = np(1-p)$ $np = (1-p)$ $np + p = 1$ $p = \dfrac{1}{n+1}$ Since $p \leq 0.01$ $\dfrac{1}{n+1} \leq 0.01$ $1 \leq 0.01n + 0.01$ $n \geq \dfrac{0.00}{0.01}$ $n \geq 99$ Therefore the smallest number of trials is 99.	2 marks – correct solution. 1 mark – finds an expression for p in terms of n.
11d	$\displaystyle\int_0^{\frac{\pi}{12}} \sin^2 4x \; dx = \int_0^{\frac{\pi}{12}} \frac{1}{2}(1 - \cos 8x)$ $= \dfrac{1}{2}\left[x - \dfrac{1}{16}\sin 8x\right]_0^{\frac{\pi}{12}}$ $= \dfrac{\pi}{24} - \dfrac{1}{32}\sin\left(\dfrac{2\pi}{3}\right)$ $= \dfrac{\pi}{24} - \dfrac{\sqrt{3}}{32}$ $= \dfrac{4\pi - 3\sqrt{3}}{96}$	2 marks – correct solution. 1 mark – correct integration.
11ei	$P(4) = 0$ $4a + b = 0$ (1) $P(-1) = 15$ $-a + b = 15$ (2) (1) – (2) : $-5a = 15$ $a = -3$ $b = 12$	2 marks – correct solution. 1 mark – obtains 2 equations for a and b.

11eii	Since Dividend = Divisor xQuotient + Reminder $P(x) = (x+1)(x-4)Q(x) - 3x + 12$ Remainder = $-3x + 12$	1 mark – correct solution
11fi	Consider the given equation $T = 36 + Ae^{-kt}$ $\frac{dT}{dt} = -kAe^{-kt}$ Since $T - 36 = Ae^{-kt}$ $\frac{dT}{dt} = -k(T-36)$	1 mark – correct solution
11fii	Initially (when $t = 0$), the temperature of the ice cream is $-18°C$. $-18 - 36 = Ae^{-0}$ $A = -54$ Also $T = -10°C$ when $t = 1$ $-10 = 36 - 54e^{-k}$ $\therefore \quad k = -\ln\left(\frac{23}{27}\right)$ To begin melting, $t = 0°C$: $0 = 36 - 54e^{\ln\left(\frac{23}{27}\right)t}$ $e^{\ln\left(\frac{23}{27}\right)t} = \frac{36}{54}$ $t = \frac{\ln\left(\frac{36}{54}\right)}{\ln\left(\frac{23}{27}\right)} = 2.5287$ $\therefore \quad t = 2.5$ minutes (to 1 d.p)	3 marks – correct solution. 2 marks – finds correct value of A and k. 1 mark - finds value of A or demonstrates equivalent understanding
12a	8 letters, 5 consonants and 3 vowels. All are distinct. Number of arrangements with all vowels together and not all consonants together is: CVVVCCCC, CCVVVCCC, CCCVVVCC, CCCCVVVC = 4 WAYS. So 4 x 3! x 5! = 2880	2 marks – correct solution 1 mark – correct procedure or equivalent progress shown.

12bi	$\sin y = 2x$ $x = \frac{1}{2}\sin y$ $\text{Area} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin y \; dy$ $= -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right]$ $= -\frac{1}{2}[0-1] = \frac{1}{2}\; units^2$	2 marks – correct solution 1 mark – writes correct integral for the shaded area
12bii	$x = \pi\int x^2 \; dy$ $= \pi\int_0^{\frac{\pi}{2}} \frac{1}{4}\sin^2 y \;\; dy$ $= \frac{\pi}{4} \times \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - \cos 2y) \; dy$ $= \frac{\pi}{8}\left[y - \frac{\sin^2 y}{2}\right]$ $= \frac{\pi}{8}\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - (0-0)\right]$ $= \frac{\pi^2}{16}\; units^3$	3 marks – correct solution 2 marks – integrates correctly 1 mark – finds an integral expression of the volume
12c	$\frac{1}{x(2-x)} \geq 1\,, \quad x \neq 0, 2$ $x(2-x) \geq (x(2-x))^2\,, \quad x \neq 0, 2$ $(x(2-x))^2 - (x(2-x) \leq 0, \quad x \neq 0, 2$ $x(2-x)(x(2-x)-1) \leq 0, \quad x \neq 0, 2$ $x(2-x)(2x-x^2-1) \leq 0, \quad x \neq 0, 2$ $x(x-2)(x^2-2x+1) \leq 0, \quad x \neq 0, 2$ $x(x-2)(x-1)^2 \leq 0, \quad x \neq 0, 2$ 	3 marks – correct solution 2 marks – correct process but does not account for x <> 0, 2 1 mark – multiplies both sides by the square of the denominator or equivalent merit.

	$f(x) \leq 0$ for $x \in [0,2]$ but $x \neq 0, 2 \therefore x \in (0,2)$	
12d	$\int_0^{\frac{1}{\sqrt{3}}} \frac{\sin(\tan^{-1}x)}{1+x^2}\,dx$ $u = \tan^{-1}x$ $du = \frac{dx}{1+x^2}$ when $x = 0$ $u = \tan^{-1}0 = 0$ when $x = \frac{1}{\sqrt{3}}$ $u = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ $\int_0^{\frac{\pi}{6}} \sin u \text{ du}$ $= -\left(\cos\frac{\pi}{6} - \cos 0\right)$ $= -\frac{\sqrt{3}}{2} + 1$	2 marks – correct solution 1 mark – correctly expresses integral in terms of u

12e	Body is stationary therefore the forces are in equilibrium (sum of horizontal components =0, sum of vertical components = 0) Horizontal: $-F_1 + 8 + F_2 \cos 60° - 6 \cos 30° = 0$ $F_1 = 8 + F_2 \cos 60° - 6 \cos 30°$ $F_1 = 8 + \frac{1}{2} F_2 - 3\sqrt{3}$ ① Vertical $F_2 \sin 60° = 6 \sin 30°$ $F_2 = \frac{6}{\sqrt{3}}$ ② sub in ① $F_1 = 8 + \frac{1}{2} \times \frac{6}{\sqrt{3}} - 3\sqrt{3}$ $= 8 + \frac{3}{\sqrt{3}} - 3\sqrt{3}$ $= 8 + \sqrt{3} - 3\sqrt{3}$ $= 8 - 2\sqrt{3}$	3 marks – correct solution 2 marks – resolves forces horizontally and vertically 1 mark – resolves forces correctly in one direction or equivalent merit.
13a	$x - 1 = \sqrt{x^2+1} \times \sqrt{2}\ \cos\theta$ (i) $1 - x = \sqrt{2}\sqrt{x^2+1}\cos\varphi$ (2) From (1) $\cos\theta = \frac{x-1}{\sqrt{2}\sqrt{x^2+1}}$ From (2) $\cos\varphi = \frac{1-x}{\sqrt{2}\sqrt{x^2+1}}$ $\cos\theta\cos\varphi = \frac{x-1}{\sqrt{2}\sqrt{x^2+1}} \times \frac{1-x}{\sqrt{2}\sqrt{x^2+1}}$ $\cos\theta\cos\varphi = \frac{x-1}{\sqrt{2}\sqrt{x^2+1}} \times \frac{-(x-1)}{\sqrt{2}\sqrt{x^2+1}}$ $\cos\theta\cos\varphi = \frac{-(x-1)^2}{2(1+x^2)}$	1 mark for expressions of $\cos\theta$ and $\cos\varphi$ 1 mark for showing final result. Generally well done. A few errors calculating the magnitudes.

13b	$p(p=0) = \frac{32}{243}$ $q^5 = \frac{32}{243}$ $q = \frac{2}{3}$ $\therefore p = \frac{1}{3}$ $P(\hat{p} > 0.6)$ $P(\hat{p}) = 1 = p^5$ $= \left(\frac{1}{3}\right)^5$ $= \frac{1}{243}$ $P(\hat{p})$ $= 0.8$ ${}^5_4 p^4 q^1 = 5 \times \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1$ $= \frac{10}{243}$ $P(\hat{p}) > 0.6 = \frac{1}{243} + \frac{10}{243}$ $= \frac{11}{243}$ $= 0.04526748......$ $= 0.0453$ (4 dp)	1 mark for value of p 1 mark for correct binomial expression 1 mark for correct answer Some didn't apply binomial probability. A few didn't express the final answer as a decimal.
13c	$\frac{dy}{\sqrt{16-3y^2}} = dx$ $\int \frac{dy}{\sqrt{16-3y^2}} = \int dx$ $\frac{1}{\sqrt{3}} \int \frac{\sqrt{3}}{\sqrt{16-3y^2}} dy = \int dx$ $\frac{1}{\sqrt{3}} sin^{-1} \frac{y\sqrt{3}}{4} = x + c$ $\frac{1}{\sqrt{3}} sin^{-1} \frac{2\sqrt{3}}{4} = c$ $\frac{1}{\sqrt{3}} sin^{-1} \frac{\sqrt{3}}{2} = c$ $\frac{1}{\sqrt{3}} \frac{\pi}{3} = c$ $\frac{\pi}{3\sqrt{3}} = c$ $x + \frac{\pi}{3\sqrt{3}} = \frac{1}{\sqrt{3}} sin^{-1} \frac{y\sqrt{3}}{4}$ $\sqrt{3}x + \frac{\pi}{3} = sin^{-1} \frac{y\sqrt{3}}{4}$ $\sin\left(\sqrt{3}x + \frac{\pi}{3}\right) = \frac{y\sqrt{3}}{4}$ $\frac{4}{\sqrt{3}} \sin\left(\sqrt{3}x + \frac{\pi}{3}\right) = y$	1 mark correct integration 1 mark correct value of c 1 mark for correct expression in terms of x Some students didn't know what to do with the $\sqrt{3}$

13d	$\cos\frac{\pi}{4} = 1 - 2\sin^2\frac{\pi}{8}$ $\frac{1}{\sqrt{2}} = 1 - 2\sin^2\frac{\pi}{8}$ $\frac{1-\sqrt{2}}{\sqrt{2}} = -2\sin^2\frac{\pi}{8}$ $\frac{1-\sqrt{2}}{\sqrt{2}\times 2} = -\sin^2\frac{\pi}{8}$ $\frac{\sqrt{2}-1}{\sqrt{2}\times 2} = \sin^2\frac{\pi}{8}$ $\pm\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}\times 2}} = \sin\frac{\pi}{8}$ But $\sin\frac{\pi}{8} > 0$ $\sin\frac{\pi}{8} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$ $\sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$	1 mark for expressing as a relevant double angle 1 mark for expression for $\sin^2\frac{\pi}{8}$ 1 mark for a final correct answer Some didn't find expression(s) in terms of $\frac{\pi}{4}$
13 e i)	The curved edge of the sphere is a circle with radius10 and centre (0,10) $x^2 + (y-10)^2 = 100$ $x^2 + y^2 - 20y + 100 - 100 = 0$ $x^2 = 20y - y^2$	"show" Needs centre and radius
13 eii)	The volume of revolution is given by $\int_0^h f(y)dy$. From part (i): $V = \pi\int_0^h x^2\,dy$ $= \pi\int_0^h (20y - y^2)\,dy$ $= \pi\left[10y^2 - \frac{1}{3}y^3\right]_0^h$ $= \frac{1}{3}\pi h^2(30-h)$	1 mark for correct integration 1 mark for correct answer This was poorly done. (Revolution around the y-axis)
e) iii)	Differentiate the expression of volume found in part (e) (ii) with respect to h :	

	$$\frac{dV}{dh} = \pi \frac{d}{dh}\left(10h^2 - \frac{1}{3}h^3\right)$$ $$\frac{dV}{dh} = \pi(20h - h^2)$$ When $h = 6$, we have: $$\frac{dV}{dh} = \pi(20 \cdot 6 - 6^2) = 84\pi$$ Given that $\frac{dV}{dt} = 20$ and by using the chain rule, it follows: $$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$ $$20 = 84\pi \times \frac{dh}{dt}$$ $$\therefore \frac{dh}{dt} = \frac{5}{21\pi} m/minute$$	1 mark for correct answer (Check for error carried forward) Many non-attempts.

14a	$x = \dfrac{2}{y^2+1}$ $y^2 = \dfrac{2}{x} - 1$ $y = \pm\sqrt{\dfrac{2}{x} - 1}$ Since $N < 0$, $$f^{-1}(f(N)) = \sqrt{\dfrac{2}{\left(\dfrac{2}{N^2+1}\right)} - 1}$$ $= \sqrt{N^2}$ $= \lvert N \rvert$ Since $N < 0$, so $$f^{-1}(f(N)) = \lvert N \rvert = -N$$	2 marks correct solution 1 mark correct arm of inverse function must be chosen after stating +/- MUST state **reason and have fully** simplified expression
14b	$f'(x) = e^{2x} + 2xe^{2x}$ $= (2x+1)e^{2x}$ $= (2^1 x + 1 \times 2^{1-1})e^{2x}$ and so the statement is true for $n = 1$. Assume that the statement is true for some $k > 1$. That is $f^{(k)}(x) = (2^k x + k2^{k-1})e^{2x}$ then $f^{(k+1)}(x) = \dfrac{d}{dx}\left((2^k x + k2^{k-1})e^{2x}\right)$ $= 2^k e^{2x} + 2(2^k x + k2^{k-1})e^{2x}$ $= (2^{k+1}x + (k+1)2^k)e^{2x}$ and so the statement is true for $n = k+1$. Therefore, by the principle of mathematical induction, the statement is true for all $n \in Z^+$.	3 marks correct solution 2 marks **ONLY** one error with no lines of working omitted and clearly showing **ALL** substitutions and algebraic manipulations 1 mark correct initial case proven with **ALL** substitutions and algebraic manipulations
14c	Using sum to products trigonometric identities: $\sin 3x = \sin 2x$ $\sin 3x - \sin 2x = 0$ $2\cos\dfrac{5x}{2}\sin\dfrac{x}{2} = 0.$ Which we can solve as $\cos\dfrac{5x}{2} = 0$ or $\sin\dfrac{x}{2} = 0$ $\dfrac{5x}{2} = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2}, \dfrac{9\pi}{2}$ or $\dfrac{x}{2} = 0, \pi$ in the domain where $x \in [0, 2\pi]$ $x = 0, \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \pi, \dfrac{7\pi}{5}, \dfrac{9\pi}{5}, 2\pi.$	3 marks correct solution. 2 marks **ONLY** one error, without **QS** for CFE 1 mark full correct solution using one valid factor of the original (max two errors **ONLY**)

14d	$n = 100, p = 0.16, q = 0.84$ $np = 16 > 10$ and $nq = 84 > 10$ $\therefore$ normal approx can be used $\mu_{\hat{p}} = 0.16$ and $\sigma_{\hat{p}} = \sqrt{\dfrac{(0.16)(0.84)}{100}} = 0.0367$ $z_{0.1} = \dfrac{0.1 - 0.16}{0.0367} = -1.6367$ $P\left(\hat{p} > 10\%\right) = P(z > -1.6367) = P(z < 1.6367)$ $= 0.9495$ (using z-score tables)	3 marks correct solution 2 marks **ONLY** one error and showing BN check, z-score calc and correct % using symmetry 1 mark **max two errors** with correct approach to calculating % using given z-score table
14ei	Factorise the trigonometric function as follows: $y = \left(\sqrt{3}+1\right)\cos 2x + \left(\sqrt{3}-1\right)\sin 2x$ Using compound angle formula: $R\cos(2x - \alpha) = R\cos\alpha\cos 2x + R\sin\alpha\sin 2x$ $R\cos\alpha = \sqrt{3}+1$ $R\sin\alpha = \sqrt{3}-1$ Hence $R^2 = 8$ $\quad \tan\alpha = 2 - \sqrt{3}$ $R = 2\sqrt{2}$ $\quad \alpha = \dfrac{\pi}{12}$ Therefore, the trigonometric function is $y = 2\sqrt{2}\cos\left(2x - \frac{\pi}{12}\right) = 2\sqrt{2}\cos 2\left(x - \frac{\pi}{24}\right)$ The maximum value (point A) is $2\sqrt{2}$ because of the amplitude of the cosine function To get the x-coordinate of the first maximum point A, using the above transformation the graph is out-of phase $\frac{\pi}{24}$ Hence, the coordinates of A are $\left(\frac{\pi}{24}, 2\sqrt{2}\right)$. To get the x-intercept,point B solve the following: $2\sqrt{2}\cos\left(2x - \dfrac{\pi}{12}\right) = 0$ $\cos\left(2x - \dfrac{\pi}{12}\right) = 0$	2 marks correct solution 1 mark max one error and no QS with correct coordinates for one point showing all working.

	$$2x-\frac{\pi}{12}=\frac{\pi}{2}$$ $$\therefore x=\frac{7\pi}{24}$$ Hence, the coordinates of B are $\left(\frac{7\pi}{24},0\right)$.	
14eii	$$2\sqrt{2}\cos\left(2x-\frac{\pi}{12}\right)=2\sqrt{2}$$ $$\cos\left(2x-\frac{\pi}{12}\right)=1$$ $$2x-\frac{\pi}{12}=0,2\pi$$ $$\therefore x=\frac{\pi}{24},\frac{25\pi}{24}$$	2 marks correct solution 1 mark ONLY one error in correct approach to solving for required values.